Domain Engineering with Concepts

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Software Product Line Benefits

Software product line development versus normal development

- Productivity improvement: up to a factor of 10
- Quality improvement: up to a factor of 10
- Decreased cost: by as much as 60%
- Decreased labour needs: by as much as 87%
- Decreased time to market: by as much as 98%
- Ability to move into new markets: in months, not years

Each of the above is based on a documented product line effort
http://www.sei.cmu.edu/library/assets/spl-essentials.pdf
Linda Northrop, 2008

Software Product Line

Also called a *product family*

- A set of software-intensive systems
- Built for a particular market segment (domain)
- Created from a common set of core assets
  - Libraries, architectures, tests, tools, project planning

Core asset development:

Domain engineering

Application development*: Application engineering*

Defining the Core Assets of a Domain

Must fit the language of software

Algorithms + Data Structures = Programs

Niklaus Wirth 1976

- A **Data Structure** *abstracts to* a **type**
  - Values of a type can be compared for **equality**
- An **Algorithm** *abstracts to* a **function**
  - Input argument list
  - Result type
- Properties of a type are defined by **predicates** on expressions

  T a,b,c;

  assert ((a+b)+c == a+(b+c));

Questions to ask of a Domain

- What are the **types**
- What are the **functions**
- What are the **axioms**

What are the (C++) **concepts**

```
template<typename m>
concept monoid (binary<m> bin, nullary<m> unit) {
  axiom associative (m a, m b, m c) {
    assert bin(bin(a,b),c) == bin(a,bin(b,c));
  }
  axiom neutral (m a) {
    assert bin(a,unit()) == a;
    assert bin(unit(),a) == a;
  }
}
```


Data Structure Algebra

Isomorphisms

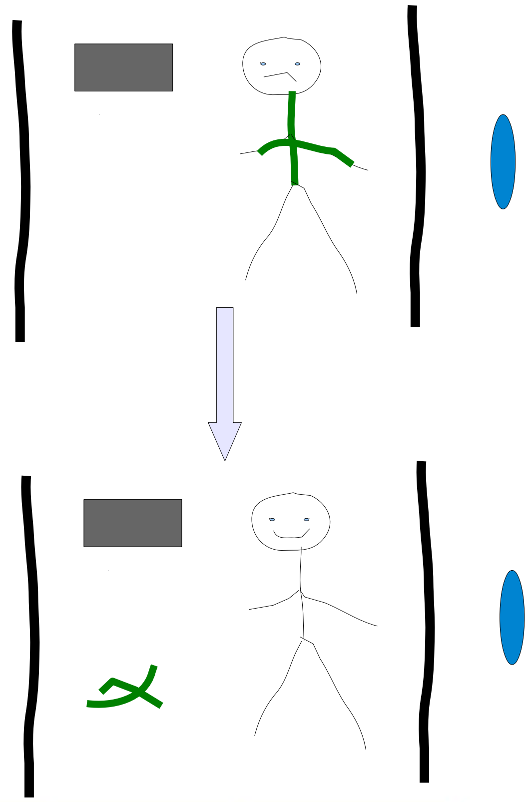
- The same information content for different declarations

```
struct {
      int a[100];
      int b[100];
} d1;
```

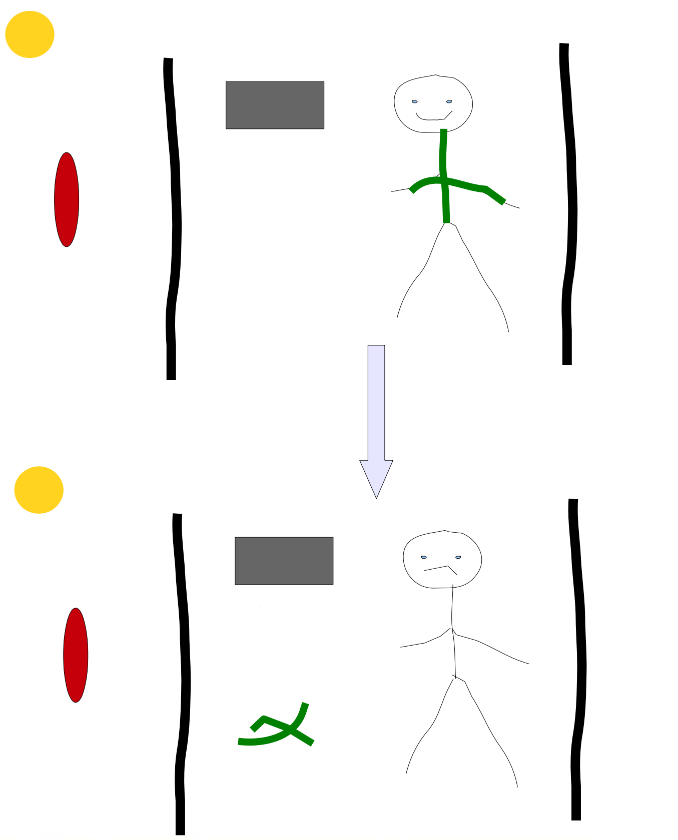
```
struct D {
      int a;
      int b;
};
D d2[100];
```

- Alternative data structures
  - Different access patterns
  - Different *abstractions*

The Heat Problem: Norway

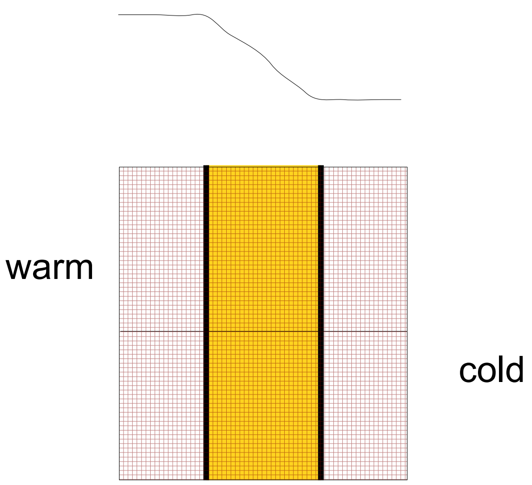


The Heat Problem: Texas



The Heat Equation

Temperature across
the wall



Grid for the wall
and surroundings

$$\frac{\partial}{\partial t} u = \alpha * (\nabla \cdot (\nabla u)) + f$$

Variables, in space and time
u – temperature, scalar field
α – thermal diffusivity, scalar field
f – heat source, scalar field

Derivatives
∂/∂t – partial derivative in time
∇ – gradient, scalar field to vector field
∇· – divergence, vector field to scalar field

Operations
* – scalar field multiplication
+ – scalar field addition

Concepts for Arithmetic Operations

```
template<typename r>
concept unit_ring(binary<r> plus, unary<r> minus, binary<r> mult) {
  axiom abelian_group(r a, r b, r c) {
    assert plus(plus(a,b),c) == plus(a,plus(b,c));
    assert plus(a,b) == plus(b,a);
    assert plus(a,r(0)) == a;
    assert plus(a, minus(a)) == r(0);
  }
  axiom monoid(r a, r b, r c) {
    assert mult(mult(a,b),c) == mult(a,mult(b,c));
    assert mult(a,r(1)) == a;
    assert mult(r(1),a) == a;
  }
  axiom distributive(r a, r b, r c) {
    assert mult(a,plus(b,c)) == plus(mult(a,b),mult(a,c));
    assert mult(plus(a,b),c) == plus(mult(a,c),mult(b,c));
  }
}
```

Engineering the PDE domain

- Data field df<r>: a value of type r at every point in space-time
  - Scalar field sf<real>, ring with pointwise +,-,* and $\partial/\partial t$, $\partial/\partial x$, ..
- Matrix matrix<r> with +,-,mm from any ring r
- Matrix field with $\nabla\cdot$, ∇
  - df<matrix<real>>
  - matrix<sf<real>>

Choosing matrix field format: consider the derivation operations

- Derivatives require access to neighbouring data
- Scalar field has partial derivatives $\partial/\partial t$, $\partial/\partial x$, ..
  - The derivations can be defined from partial derivatives

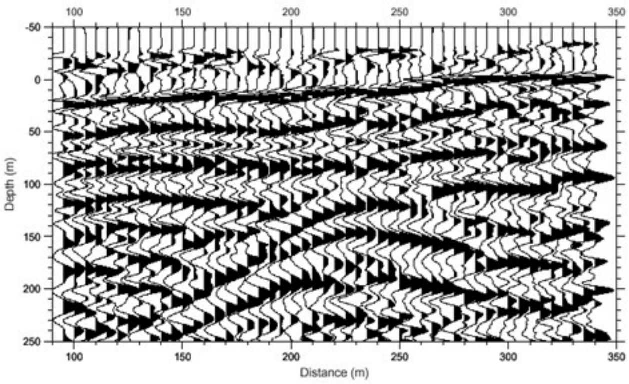
Dot Product Problem

```
template<typename r> r dot(vector<r> a, vector<r> b) {
  return ∑_i a[i] * b[i];
}
template<typename r> vector<r> new_coordinate( matrix<r> m, vector<r> v) {
  return mm(m,v);
}

template<typename r>
concept dot_properties () {
  axiom coordinate_system_invariance(matrix<r> m, vector<r> u, vector<r> v) {
    assert dot(u,v) == dot(new_coordinate(m,u),new_coordinate(m,v));
  }
  // ...
}
```

- Dot algorithm is wrong? Take coordinate system into account
- Typing is wrong? Vector and covector
- Change of coordinate algorithm is wrong? Covectors are different

Seismic Waves



$$\rho \frac{\partial}{\partial t} \frac{\partial}{\partial t} u = \nabla \cdot \sigma + f \,,$$
$$\sigma = \Lambda \circ e \,,$$
$$e = L(u, g)$$

Elastic wave equation

Variables
ρ – density, scalar field
u – displacement, vector field
σ – stress, matrix field
f – external force, vector field
Λ – stiffness, tensor field
e – strain, matrix field
g – metric, matrix field

Derivatives
∂/∂t – partial derivative in time
∇· – divergence, matrix field to vector field
L – Lie derivative, matrix field to matris field

Operations
∘ – tensor application, returns matrix field
\+ – vector field addition

Conclusions

- Domain engineering
  - Defines the core assets of a software domain
  - Essential for software product lines
  - Precedes application engineering
- C++ style concepts for core asset development
  - Libraries
    - Declares types, declares functions, defines axioms
    - Drives towards a comprehensive API
  - Architectural considerations
  - Testing
    - Axioms as test oracles
  - Tools: refactoring and optimisation
    - Equational axioms as refactoring rules