Putting Polyhedral Loop Transformations to Work

Unified Model and Compiler Interface

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PRISM

Research Context

- Whole program optimization for peak performance
  - Uniprocessor
  - OpenMP
- Iterative, feedback-directed optimization
  1. Implement the *useful* transformations
  2. Choose the transformation *sequence and parameters*

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- This talk: *promote the polytope model as a viable representation and transformation framework for semi-automatic program optimization and parallelization*

Example: Matrix Multiplication

Alpha EV67, dynamic analysis with Compaq's Alpha simulator

$\mathbf{95\%}$ of peak performance (Parello and Temam, SuperComputing'02)

Transformations	*Speedup*
Original `ikj` loop nest, Compaq f90 -O2 -unroll 1 -nopipeline	**1.00**
Compaq f90 -O5 + KAP	**3.37**
3D blocking for L1 and TLB	2.62
3D + *interchange for store queue* + *unrolling for ILP*	**3.71**
3D + int + unroll + *register blocking*	9.90
3D + int + unroll + reg block + *prefetch*	**10.37**
3D for L1 and L2 + *copy for TLB* + int + unroll + reg block + prefetch	12.75
3D + copy + int + unroll + reg block + prefetch + *low level opt*	**13.56**

Example of Composition of Transformations

```
for (i=0; i<1000; i++)
  for (j=0; j<m; j++)
    B[j] = A[i][j] + …
  for (j=0; j<n; j++)
    … = B[j] + …
```

$\xrightarrow{\textit{fuse}}$

```
for (i=0; i<1000; i++)
  for (j=0; j<max(m,n); j++)
    if (j<m)
      B[j] = A[i][j] + …
    if (j<n)
      … = B[j] + …
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    if (j<n)
      … = B[j] + …
```

$\xrightarrow{\textit{shift}}$

```
for (i=0; i<1000; i++)
  for (j=0; j<max(m,n+1); j++)
    if (j<m)
      B[j] = A[i][j] + …
    if (j>0 && j<=n)
      … = B[j-1] + …
```

Example of Composition of Transformations

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for (i=0; i<1000; i++)
  for (j=0; j<max(m,n+1); j++)
    if (j<m)
      B[j] = A[i][j] + …
    if (j>0 && j<=n)
      … = B[j-1] + …
```

$\xrightarrow{\textit{strip-mine}}$

```
for (ii=0; ii<1000; ii+=10)
  for (i=ii; i<ii+10; i++)
    for (j=0; j<max(m,n+1); j++)
      if (j<m)
        B[j] = A[i][j] + …
      if (j>0 && j<=n)
        … = B[j-1] + …
```

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for (ii=0; i<1000; i+=10)
  for (i=ii; i<ii+10; i++)
    for (j=0; j<max(m,n+1); j++)
      if (j<m)
        B[j] = A[i][j] + …
      if (j>0 && j<=n)
        … = B[j-1] + …
```

$\xrightarrow{\textit{prefetch}}$

```
for (ii=0; i<1000; i+=10)
  for (i=ii; i<ii+10; i++)
    for (j=0; j<max(m,n+1); j++)
      if (j<m)
        if (j%4==0)
          prefetch A[i+1][j]
        B[j] = A[i][j] + …
      if (j>0 && j<=n)
        … = B[j-1] + …
```

Some Problems With Syntax-Based Approaches

- Control overhead
  - Regenerate control structures after each transformation
  - Fixed transformation sequence
  - Non-local transformations

```
if (m<n && m%4==0)
  for (ii=0; i<1000; i+=10)
    for (i=ii; i<ii+10; i++)
      for (j=0; j+3<m; j+=4)
        prefetch A[i+1][j]
        B[0] = A[i][j] + …
        … = B[j-1] + …
        B[j] = A[i][j+1] + …
        … = B[j] + …
        B[j] = A[i][j+2] + …
        … = B[j+1] + …
        B[j] = A[i][j+3] + …
        … = B[j+2] + …
      for (j=m; j<n; j++)
        … = B[j-1] + …
else if (m<n && m%4==1)
  …
```

1. POLYHEDRAL REPRESENTATION

Unified Loop Nest Transformation Framework

- Operated by optimization *and* architecture *experts*

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- Express any *composition* of analyses and transformations
- Domain-specific, representation of *loop nests*
- No *intermediate* translation to syntax-tree

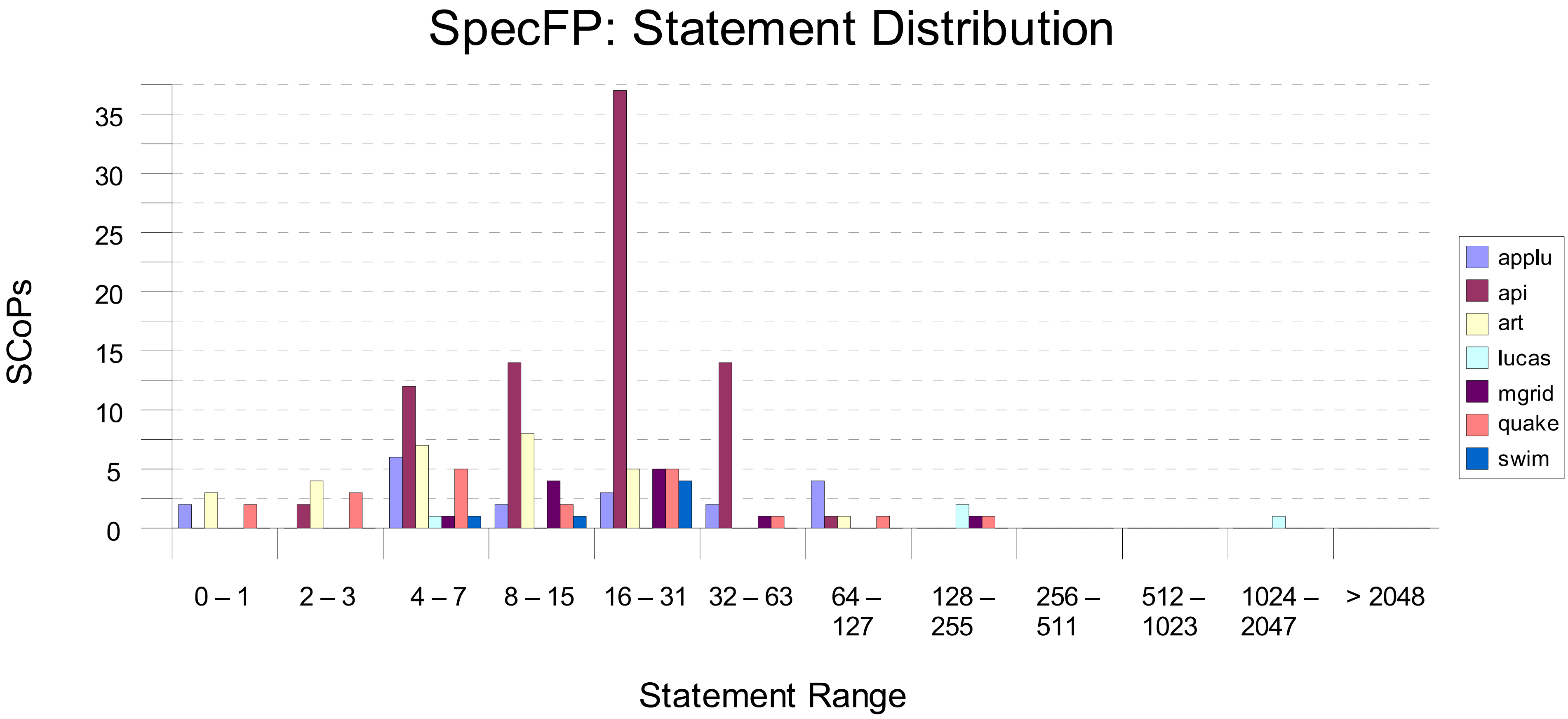
Static Control Parts (SCoPs)

```
     for (i=1; i<3; i++)
 ..........................................................................
 S1  | ...                                  | SCoP 1, one statement
 ..........................................................................
     | while (A[j]!=0)
 ..........................................................................
 S2  | | ...                                | SCoP 2, three statements
     | | for (k=0; k<j; k++)                |   parameters: i,j
     | | | if (j>=2)                        |   iterators: k
 S3  | | | | ...                            |
 S4  | | | ...                              |
 ..........................................................................
     | for (p=0; p<6; p++)                  | SCoP 3, two statements
 S5  | | ...                                |   iterators: p
 S6  | | ...                                |
```

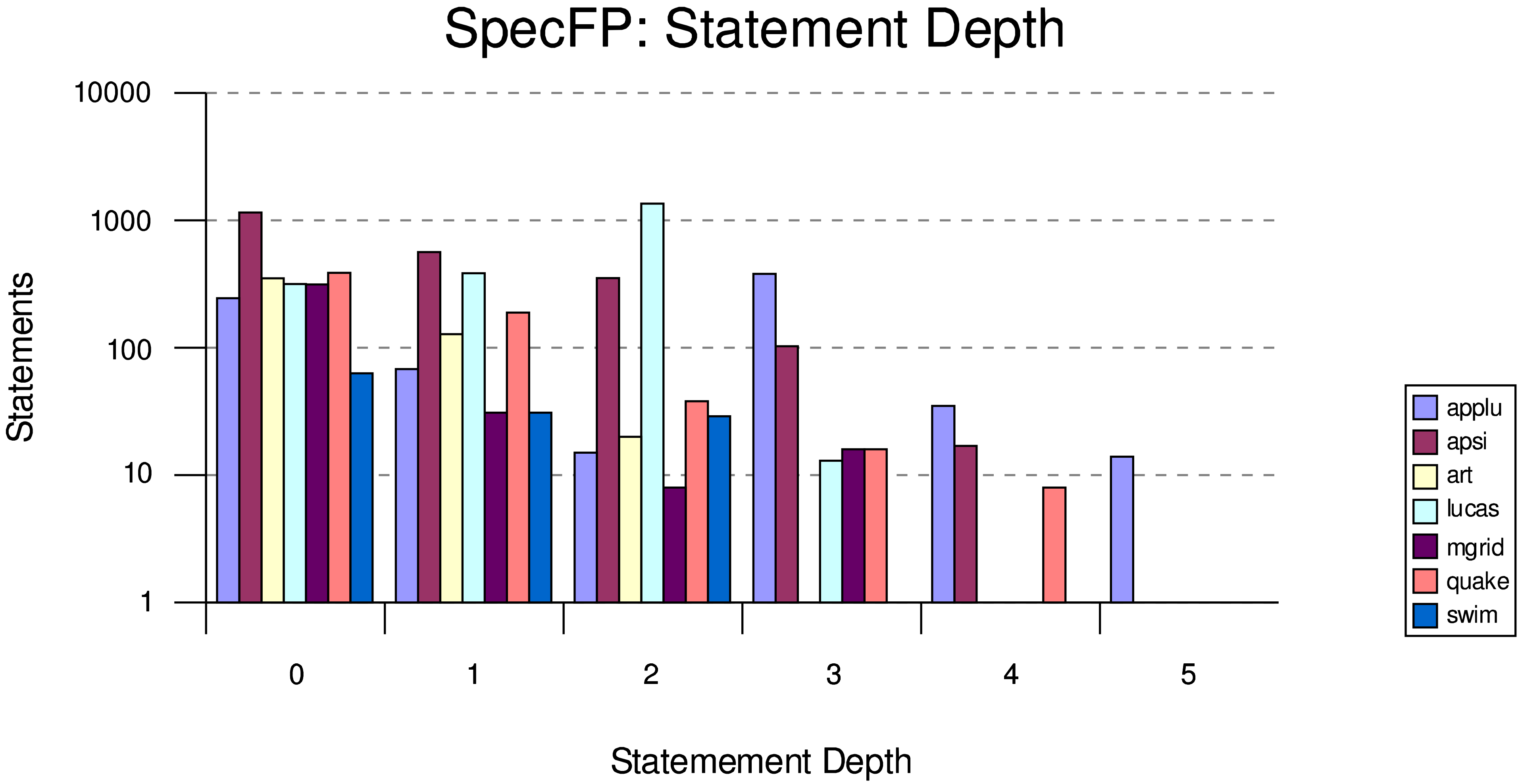
SCoP Coverage (SpecFP)

	SCoPs			Statements		Array References	
	All	Param.	`ifs`	All	in SCoPs	All	Affine
applu	19	15	1	757	84%	1245	100%
apsi	80	80	25	2192	84%	977	78%
art	28	27	4	499	69%	52	100%
lucas	4	4	2	2070	99%	411	40%
mgrid	12	12	2	369	100%	176	99%
quake	20	14	4	639	77%	218	100%
swim	6	6	1	123	100%	192	100%

SCoP Size (SpecFP)



SCoP Depth (SpecFP)



SCoP Polyhedral Representation

- Describes each statement *separately*
- Captures *control* and *array access* semantics

  Through *parameterized* affine (in)equalities

  1. A *domain*

     The bounds of the enclosing loops

  2. A *schedule*

     An affine function assigning logical dates to iterations

  3. A list of *access functions*

     To describe array references

Existing Polyhedral Representations

- A few facts

1. Polytopes are very expressive

   $\boldsymbol{AX \geq 0}$ suffices to characterize all executions of a statement (schedule, domain and memory accesses)

2. Affine schedules emerged in automatic parallelization
3. Affine schedules are not popular for optimization

   (too expensive, too restrictive, too general, non intuitive...)

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- Some common biases
  - ? Schedules are only meant to describe parallelism
  - ? Only one-dimensional schedules are useful
  - ? Schedule and domains can be merged in one matrix

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- Some common biases
  - ? Schedules are only meant to describe parallelism
  - ? Only one-dimensional schedules are useful
  - ? Schedule and domains can be merged in one matrix
- We use *separate* matrices and *full*-dimensional *sequential* schedules

Affine Schedule

- Dense, totally ordered (sequential) schedule
- Unimodular matrix for iteration ordering
- Matrix for parameterization and iteration shifting
- Vector for instruction scattering

Affine Schedule

- Dense, totally ordered (sequential) schedule
- Unimodular matrix for iteration ordering $(\mathbf{A})$
- Matrix for parameterization and iteration shifting $(\boldsymbol{\Gamma})$
- Vector for instruction scattering $(\boldsymbol{\beta})$

$$\boldsymbol{\theta}(\vec{\imath}, \vec{q}) = \begin{bmatrix} 0 & \cdots & 0 & 0 & \cdots & 0 & \beta_0 \\ A_{1,1} & \cdots & A_{1,d} & \Gamma_{1,1} & \cdots & \Gamma_{1,g} & \Gamma_{1,g+1} \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \beta_1 \\ A_{2,1} & \cdots & A_{2,d} & \Gamma_{2,1} & \cdots & \Gamma_{2,g} & \Gamma_{2,g+1} \\ \vdots & \vdots & \vdots & 0 & \cdots & 0 & \vdots \\ A_{d,1} & \cdots & A_{d,d} & \Gamma_{d,1} & \cdots & \Gamma_{d,g} & \Gamma_{d,g+1} \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \beta_d \end{bmatrix} \begin{bmatrix} i_1 \\ \vdots \\ i_d \\ q_1 \\ \vdots \\ q_g \\ 1 \end{bmatrix}$$

Domain and Access Functions

- Domain matrix

  Exact characterization of the valid iteration vectors

  Parameterized by symbolic constants

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- Access function

  Iteration Vector $\mapsto$ (Array Name, Vector)

  Parameterized by symbolic constants

Polyhedral Representation Example

```
      for (i=0; i<m; i++)
$S_1$   | ...
        | for (j=5; j<n; j++)
$S_2$   |   | ...
$S_3$   |   | A[2*i][j+1] = ...
```

Access function
for A[2*i][j+1]

$$\left[\begin{array}{ccccc} \mathtt{i} & \mathtt{j} & \mathtt{m} & \mathtt{n} & \mathtt{1} \\ \hline 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

2-dimensional domain of S_3
(with parameters m and n)

$$\left[\begin{array}{ccccc} \mathtt{i} & \mathtt{j} & \mathtt{m} & \mathtt{n} & \mathtt{1} \\ \hline 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & -1 & 0 & 1 & -1 \end{array}\right] \geq 0$$

5-dimensional schedule for S_3
$(\mathtt{i},\mathtt{j}) \mapsto (p_0, p_1, p_2, p_3, p_4)$

$$\left[\begin{array}{cccccccccc} p_0 & p_1 & p_2 & p_3 & p_4 & \mathtt{i} & \mathtt{j} & \mathtt{m} & \mathtt{n} & \mathtt{1} \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \end{array}\right] = 0$$

2. UNIFIED TRANSFORMATION MODEL

Primitives

Syntax	Prerequisites	Effect
RIGHTU$(\boldsymbol{S}, \mathsf{U})$	$\lvert\det(\mathsf{U})\rvert = 1$	$\mathbf{A}^{S} \leftarrow \mathbf{A}^{S}.\mathsf{U}$
SHIFT$(\boldsymbol{S}, \mathrm{M})$		$\Gamma^{S} \leftarrow \Gamma^{S} + \mathrm{M}$
FUSE$(\boldsymbol{P}, \boldsymbol{o})$		$b = \max\{\beta^{S}_{\dim(P)+1} \mid (P, o) \sqsubseteq \beta^{S}\} + 1;$ Move$((P, o+1), (P, o+1), b);$ Move$(P, (P, o+1), -1)$

Composition of Primitives

Syntax	Prerequisites	Effect
RightU(S, U)	$\lvert \det(\mathsf{U}) \rvert = 1$	$\mathbf{A}^S \leftarrow \mathbf{A}^S.\mathsf{U}$
Shift(S, M)		$\Gamma^S \leftarrow \Gamma^S + \mathrm{M}$
Fuse(P, o)		$b = \max\{\beta^S_{\dim(P)+1} \mid (P, o) \sqsubseteq \beta^S\} + 1;$ Move$((P, o + 1), (P, o + 1), b);$ Move$(P, (P, o + 1), -1)$
Tile(S, o, k) *Tiling*	$S \in \mathcal{S}$ $\wedge\, o < d^S$ $\wedge\, k > 0$	$S \leftarrow$ StripMine$(S, o, k);$ $S \leftarrow$ StripMine$(S, o + 2, k);$ $S \leftarrow$ Interchange$(S, o + 1)$

Transformation Language

- Script “generative” language
  - To produce the implementation of primitives
  - To compose primitives

Transformation Language

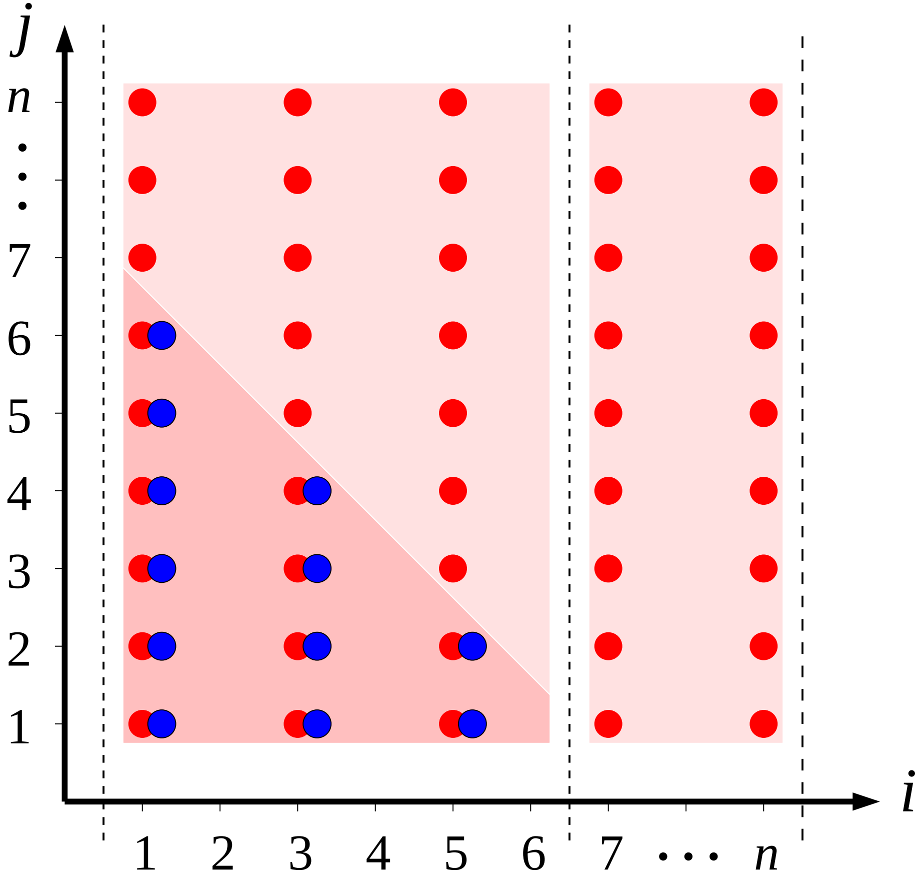
- Script “generative” language
  - To produce the implementation of primitives
  - To compose primitives
- Benefits
  - Regenerate the syntax tree *after the last transformation*
  - Few ordering constraints
  - Complex optimizations, e.g., forward array substitution
  - Combined transformations to reduce search space

    Example, “smart” register tiling: strip-mining + privatization for permutability + interchange + array contraction + register promotion

3. SOFTWARE TOOLS

Code Generation with CLooG

- Robust version of Quilleré and Rajopadhye’s algorithm
  - Parameterized unions of linearly bounded lattices
  - Depth recursion with direct optimization of conditionals
  - Tradeoff between code expansion and control overhead



```
for (i=1; i<=6; i+=2)
  for (j=1; j<=7-i; j++)
    S1; S2
  for (j=8-i; j<=n; j++)
    S1
for (i=7; i<=n; i+=2)
  for (j=1; j<=n; j++)
    S1
```

Implementation Within Open64/ORC

- *WRaP*: WHIRL Represented as Polyhedra
  - Syntax tree of *static control parts* → tree of polyhedral representations
  - Mapping polytopes to the syntax tree
    - From matrix columns to symbol table entries
    - From abstract arrays to symbol table entries
    - From abstract statements to statement nodes

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- Enables *whole program* optimization
  - Combined transformations of loops and syntactic expressions may be applied to the whole WRaP
  - Array regions, interprocedural analysis
  - Correctness and compatibility with non-affine sections

WRaP-IT: an Open64/ORC Interface-Tool

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4. *WLooG*: code generator (CLooG) with WHIRL output
   - Generate new loops, conditionals and variables
   - Move/duplicate the original statement nodes

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   - Generate new loops, conditionals and variables
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5. *Resume* the compilation flow, redoing scalar optimization

Some Related Works

- Codesign and synthesis of specialized coprocessors
  - MMAlpha
  - PICO

- Analysis and transformation frameworks
  - Omega/Petit
  - PIPS, Polaris, SUIF
  - Stratego

- Generative programming
  - ATLAS (BLAS library generator)
  - FFTW (FFT algorithm customization and optimization)
  - SPIRAL (signal-processing language, customization and optimization)

4. THANK YOU

HTTP://WWW-ROCQ.INRIA.FR/A3/WRAP-IT