Visual Servo
...through the Pages of the Transactions on Robotics (... and Automation)

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Visual Servo Control — The Basic Idea

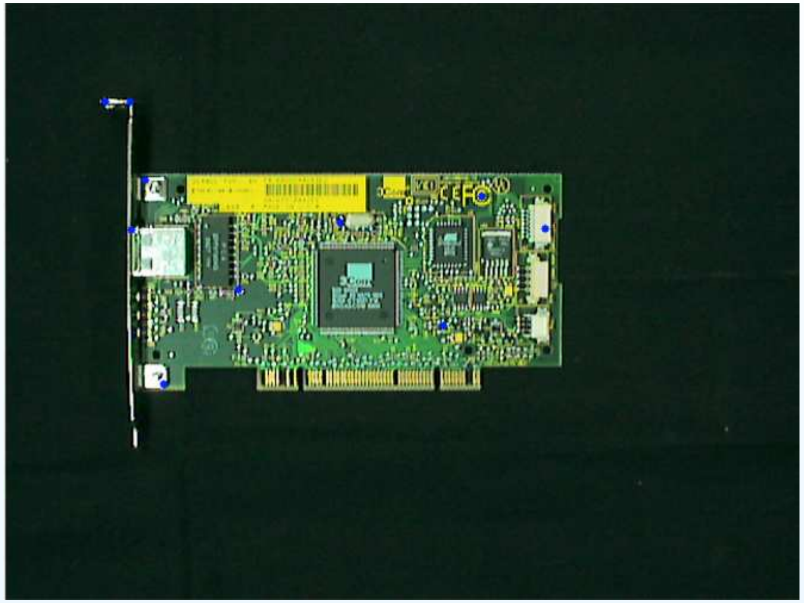
The aim of vision-based control schemes is to minimize an error $e(t)$ which is typically defined by

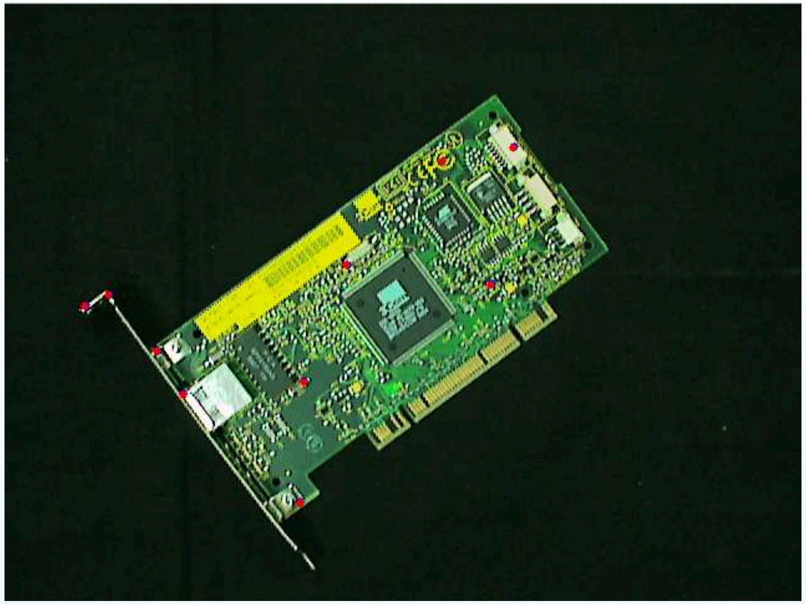
$$e(t) = s(m(t), a) - s^*$$

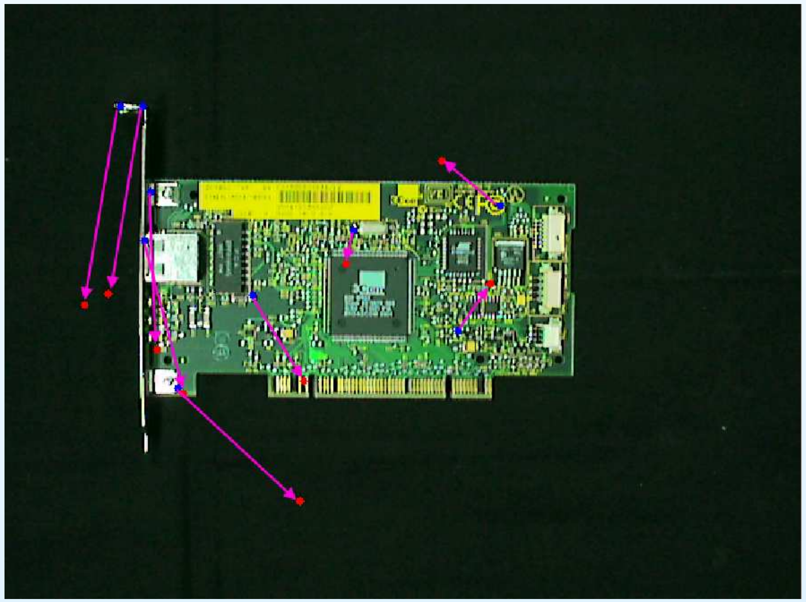
- The vector $m(t)$ is a set of image measurements (e.g., the image coordinates of interest points, or the parameters of a set of image segments).
- The image measurements are used to compute a vector of k visual features, $s(m(t), a)$.
- The vector a is a set of parameters that represent potential additional knowledge about the system (e.g., coarse camera intrinsic parameters or 3D model of objects).
- The vector s^* contains the desired values of the features.

Typically, one merely writes: $e(t) = s(t) - s^*$

An Example







$s(t) =$ coordinates of image points

The Basic Problem

There are numerous considerations when designing a visual servo system, but the basic, prototypical problem includes the following basic assumptions:

- Eye-in-hand systems — the camera is mounted on the end effector of a robot and treated as a free-flying object with configuration space $\mathcal{Q} = SE(3)$.
- Static (i.e., motionless) targets.
- Purely kinematic systems — we do not consider the dynamics of camera motion, but assume that the camera can execute accurately the applied velocity control.
- Perspective projection — the imaging geometry can be modelled as a pinhole camera.

Some or all of these may be relaxed as one progresses to more advanced topics.

Designing the Control Law — The Basic Idea

Given s, control design can be quite simple.

A typical approach is to design a velocity controller, which requires the relationship between the time variation of s and the camera velocity.

- Let the spatial velocity of the camera be denoted by $\xi = (v, \omega)$,
  - v is the instantaneous linear velocity of the origin of the camera frame and
  - ω is the instantaneous angular velocity of the camera frame.
- The relationship between $\dot{s}$ and ξ is given by

$$\dot{s} = L_s \xi$$

  in which $L_s \in \mathbb{R}^{k \times 6}$ has been called by many names, including *feature sensitivity matrix*, *interaction matrix*, and *image Jacobian*.

The key to visual servo — choosing s and the control law.

Image-Based Visual Servo Control

For Image-Based Visual Servo (IBVS)

- Features $s(t)$ are extracted from computer vision data.
- Camera pose is *not* explicitly computed.
- The error is defined in the image feature space, $e(t) = s(t) - s^*$.
- The control signal $\xi = (V, \Omega)$ is again a camera body velocity specified w.r.t. the camera frame, but for IBVS it is computed directly using $s(t)$.

For example, if the feature is a single image point with image plane coordinates u and v, we have $s(t) = (u(t), v(t))$.

A First Visual Servo System... in simulation

Weiss, L.; Sanderson, A.; Neuman, C. "Dynamic sensor-based control of robots with visual feedback," J-RA, vol.3, no.5, pp.404-417, Oct. 1987

$$\delta f_{\mathsf{ref}} = J_{\mathsf{feat}} \delta X_{\mathsf{rel}}$$

- Simulation studies (maybe not *the* first visual servo system
- Used MRAC, thus avoiding explicit computation (and derivation) of the *feature sensitivity matrix*, J_{feat}
- Observed that the ideal feature sensitivity matrix would be constant, diagonal matrix
- Introduced notion of Image-Based vs. Position-Based Visual Servo

A First Visual Servo System

Feddema, J.T.; Mitchell, O.R. "Vision-guided servoing with feature-based trajectory generation," T-RA, vol.5, no.5, pp.691-700, Oct. 1989

$$\dot{f} = {}^{f}J_c({}^{c}x)\,{}^{c}\dot{x} \quad \text{and} \quad {}^{f}J_c({}^{c}x) = \frac{\partial I}{\partial {}^{c}x}$$

- Actual experiments using PUMA robot arm (binary planar patterns)
- Loads of special-purpose vision hardware
- Trajectory planning (i.e., set points) in the image
- Controlled position and orientation about optical axis (four dof)
- Closed form for ${}^{f}J_c({}^{c}x)$

What about the control law...

Control design relies on the relationship $\dot{e} = L_e \xi$, which relates the change in error the commanded camera velocity.

Using

$$e(t) = s(t) - s^* \quad and \quad \dot{s} = L_s \xi$$

we can easily obtain the relationship between the camera velocity and the rate of change of the error, $\dot{e} = L_e \xi$ as

$$\dot{e}(t) = \dot{s}(t) = L_s \xi$$

assuming that s^* is constant.

In this case, we have $L_e = L_s \longrightarrow$ the relationship between ξ and $\dot{s}$ is the same as between ξ and $\dot{e}$.

Now our problem is merely to find the control input $\xi = u(t)$ that gives the desired error performance.

Designing the Control Law (cont)

- In many cases, we would like to ensure an exponential decoupled decrease of the error

$$\dot{e} = -\lambda e \longrightarrow L_s \xi = -\lambda e$$

- To this end, we may choose the control law as

$$u(t) = \xi = -\lambda L_e^+ e \tag{1}$$

where $L_e^+ \in \mathbb{R}^{6\times k}$ is chosen as the Moore-Penrose pseudo-inverse of L_e,

$$L_e^+ = (L_e^\top L_e)^{-1} L_e^\top$$

when L_e is of full rank 6.

- This is a *left* pseudoinverse, since

$$L_e^+ L_e = \left[(L_e^\top L_e)^{-1} L_e^\top\right] L_e = I$$

Designing the Control Law (cont)

- The choice of $\xi = -\lambda L_e^+ e$ is the usual least squares solution, and gives

$$\dot{e} = L_e \xi = -\lambda L_e L_e^+ e$$

- This is also the solution that locally minimizes $\|\xi\|$.

- When $k = 6$, if $\det L_e \neq 0$ it it possible to invert L_e. In this case we may use the control

$$\xi = -\lambda L_e^{-1} e$$

  which gives exactly the desired behavior of $\dot{e} = -\lambda e$.

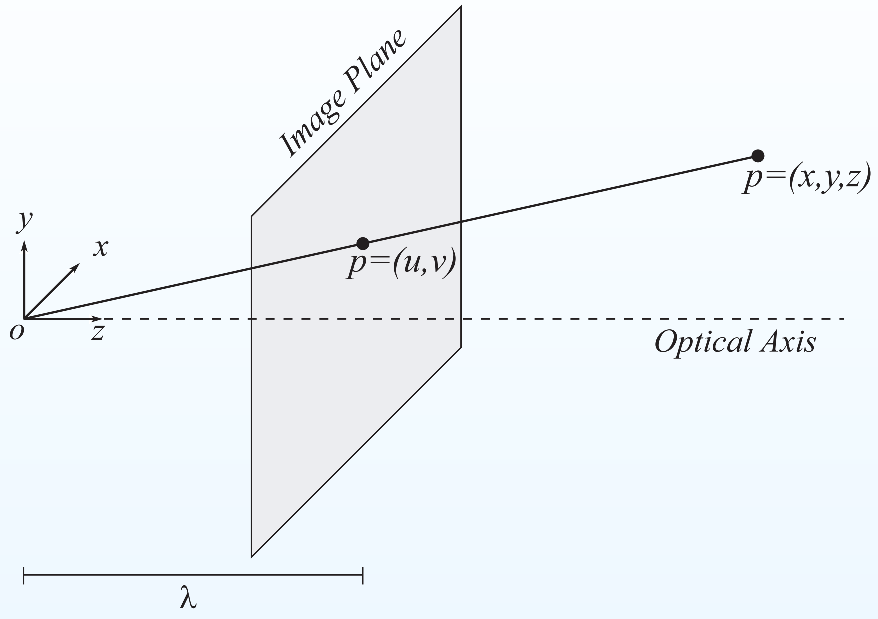
Feddema returns - with two more dof's

Feddema, J.T.; Lee, C.S.G.; Mitchell, O.R. "Weighted selection of image features for resolved rate visual feedback control," T-RA*, vol.7, no.1, pp.31-47, Feb 1991*

- Real experiments (same set-up as before)
- Full six-dof control
- Derivation of now-famous interaction matrix for points
- Added bonus of feature selection (using, e.g., condition number of J)

Nearly every paper written on the subject of visual servo control includes at least a tip of the hat to this derivation (if not a re-derivation, or restatement of the result).

Imaging Geometry



Consider a point P with coordinates (x, y, z) w.r.t. the camera frame. Using perspective projection, P's image plane coordinates are given by

$$u = \lambda \frac{x}{z}, \quad v = \lambda \frac{y}{z}$$

in which λ is the camera focal length.

The Interaction Matrix (for a point feature)

As an example, consider the interaction matrix for a single point with coordinates x, y, z.

To determine the interaction matrix for a point,

1. Compute the time derivatives for u and v.
2. Express these in terms of u, v, $\dot{x}, \dot{y}$, and $\dot{z}$ and z.
3. Find expressions for $\dot{x}, \dot{y}$, and $\dot{z}$ in terms of ξ and x, y, z.
4. Combine equations and grind through the algebra.

The Interaction Matrix (for a point feature)

Step 1:
Using the quotient rule

$$\dot{u} = \lambda \frac{z\dot{x} - x\dot{z}}{z^2}, \qquad \dot{v} = \lambda \frac{z\dot{y} - y\dot{z}}{z^2}$$

Step 2:
The perspective projection equations can be rewritten to give expressions for x and y as

$$x = \frac{uz}{\lambda}, \quad y = \frac{vz}{\lambda}$$

Substitute these into the equations above for $\dot{u}$ and $\dot{v}$.

$$\dot{u} = \lambda \frac{\dot{x}}{z} - \frac{u\dot{z}}{z}, \quad \dot{v} = \lambda \frac{\dot{y}}{z} - \frac{v\dot{z}}{z}$$

The Interaction Matrix (cont.)

Step 3:
The velocity of (the fixed point) P relative to the camera frame is given by

$$\dot{P} = -\Omega \times P - V$$

which gives equations for each of $\dot{x}$, $\dot{y}$, and $\dot{z}$.

Expanding $\dot{P} = -\Omega \times P - V$ we obtain

$$\begin{aligned}
\dot{x} &= -v_x - \omega_y z + \omega_z y \\
\dot{y} &= -v_y - \omega_z x + \omega_x z \\
\dot{z} &= -v_z - \omega_x y + \omega_y x
\end{aligned}$$

Now it's just algebra...

The Interaction Matrix (cont.)

Step 4:

Combining equations we obtain

$$\begin{aligned}
\dot{u} &= -\frac{\lambda}{z}v_x + \frac{u}{z}v_z + \frac{uv}{\lambda}\omega_x - \frac{(\lambda^2 + u^2)}{\lambda}\omega_y + v\omega_z \\
\dot{v} &= -\frac{\lambda}{z}v_y + \frac{v}{z}v_z + \frac{(\lambda^2 + v^2)}{\lambda}\omega_x - \frac{uv}{\lambda}\omega_y - u\omega_z
\end{aligned}$$

These equations can be nicely written in matrix form.

The Interaction Matrix (cont.)

In matrix form we obtain

$$\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -\frac{\lambda}{z} & 0 & \frac{u}{z} & \frac{uv}{\lambda} & -\frac{\lambda^2 + u^2}{\lambda} & v \\ 0 & -\frac{\lambda}{z} & \frac{v}{z} & \frac{\lambda^2 + v^2}{\lambda} & -\frac{uv}{\lambda} & -u \end{bmatrix} \xi$$

which can be written more compactly as

$$\dot{s} = L(s, z)\xi.$$

Feddema, et al. simply called this the *Jacobian*.

It has since been called the *Interaction Matrix* [Espiau, et al., 1992], or the *image Jacobian* [Hutchinson, et al., 1996].

The Null Space of the Interaction Matrix

The null space of this image Jacobian matrix is spanned by

$$\begin{bmatrix} u \\ v \\ \lambda \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ u \\ v \\ \lambda \end{bmatrix} \quad \begin{bmatrix} uvz \\ -(u^2+\lambda^2)z \\ \lambda vz \\ -\lambda^2 \\ 0 \\ u\lambda \end{bmatrix} \quad \begin{bmatrix} \lambda(u^2+v^2+\lambda^2)z \\ 0 \\ -u(u^2+v^2+\lambda^2)z \\ uv\lambda \\ -(u^2+\lambda^2)z \\ u\lambda^2 \end{bmatrix}$$

The Null Space of the Interaction Matrix (cont)

Intuitively, this basis of the null space corresponds to

- translation along a projection ray
- rotation about a projection ray
- rotation about the camera y-axis, keeping the camera pointed in the right direction using the linear motion
- translation in the camera y-direction, keeping the camera pointed in the right direction using the rotational motion

These are point motions that cannot be “seen" by the camera.

It seems like one should be able to exploit these “invisible" degrees of freedom...

A New Approach...

Espiau, B.; Chaumette, F.; Rives, P. "A new approach to visual servoing in robotics," T-RA, vol.8, no.3, pp.313-326, Jun 1992

- A rigorous mathematical framework for visual servo control (*c'est toujours comme ça, chez les françaises*)
- Task function approach for incorporating multiple tasks (essentially projection of secondary task onto the null space of the primary task's interaction matrix)
- Derivation of interaction matrices for several primitives (including lines, circles, spheres, etc.)
- Real experiments with 6-dof robot
- Birth of "the French school" of visual servo contro

The few other pre-1996 VS papers

- Andersson, R.L. "Dynamic sensing in a ping-pong playing robot," *T-RA*, vol.5, no.6, pp.728-739, Dec 1989
- Skaar, S.B.; Yalda-Mooshabad, I.; Brockman, W.H. "Nonholonomic camera-space manipulation," *T-RA*, vol.8, no.4, pp.464-479, Aug 1992
- Papanikolopoulos, N.P.; Khosla, P.K.; Kanade, T. "Visual tracking of a moving target by a camera mounted on a robot: a combination of control and vision," *T-RA*, vol.9, no.1, pp.14-35, Feb 1993
- Allen, P.K.; Timcenko, A.; Yoshimi, B.; Michelman, P. "Automated tracking and grasping of a moving object with a robotic hand-eye system," *T-RA*, vol.9, no.2, pp.152-165, Apr 1993
- Castano, A.; Hutchinson, S. "Visual compliance: task-directed visual servo control," *T-RA*, vol.10, no.3, pp.334-342, Jun 1994
- Fox, A.; Hutchinson, S. "Exploiting visual constraints in the synthesis of uncertainty-tolerant motion plans," *T-RA*, vol.11, no.1, pp.56-71, Feb 1995
- Yoshimi, B.H.; Allen, P.K. "Alignment using an uncalibrated camera system," *T-RA*, vol.11, no.4, pp.516-521, Aug 1995
- Papanikolopoulos, N.P.; Nelson, B.J.; Khosla, P.K. "Six degree-of-freedom hand/eye visual tracking with uncertain parameters," *T-RA*, vol.11, no.5, pp.725-732, Oct 1995

Some Highlights

- A very cool ping-pong playing robot - Andersson, 1989
  ... even though it might not really be visual servo
- Vision-based control of a nonholonomic mobile robot - Skaar, et al., 1992
- Application of state-space control methods to derive image-plane control laws (optimal, adapative) - Papanikolopoulos, et al., 1993, 1995
- A partitioned position-vision controller and a path planner for the system - Castano, Fox, Hutchinson, 1994, 1995
- Uncalibrated visual servo by exploting rotational invariance - Yoshimi, Allen, 1995

A Special Issue (well... section...) - Vol.12, no. 5, 1996

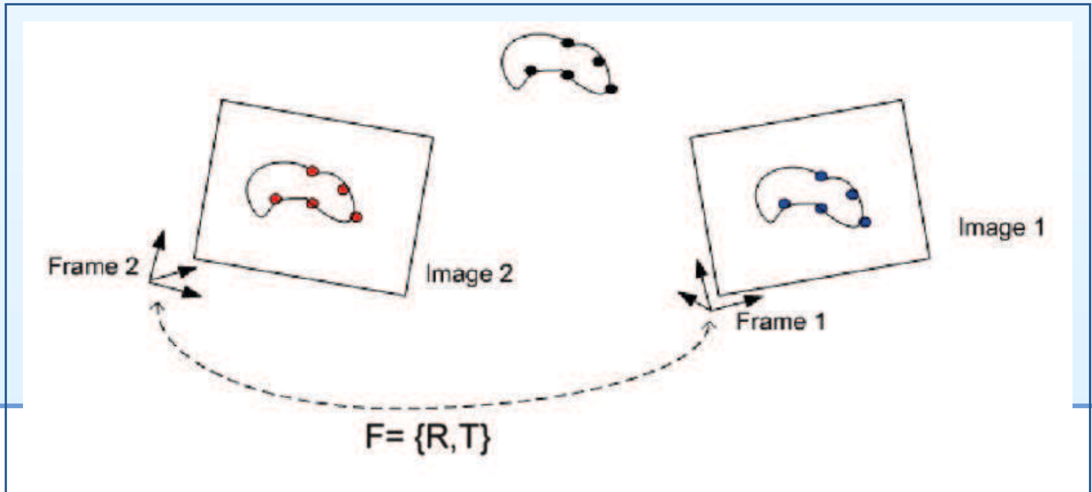
- Hutchinson, S.; Hager, G.D.; Corke, P.I. "A tutorial on visual servo control," pp.651-670
- Corke, P.I.; Good, M.C. "Dynamic effects in visual closed-loop systems," pp.671-683
- Wilson, W.J.; Williams Hulls, C.C.; Bell, G.S. "Relative end-effector control using Cartesian position based visual servoing," pp.684-696
- Rizzi, A.A.; Koditschek, D.E. "An active visual estimator for dexterous manipulation," pp.697-713
- Nelson, B.J.; Khosla, P.K. "Force and vision resolvability for assimilating disparate sensory feedback," pp.714-731
- Grosso, E.; Metta, G.; Oddera, A.; Sandini, G. "Robust visual servoing in 3-D reaching tasks," pp.732-742
- Khadraoui, D.; Motyl, G.; Martinet, P.; Gallice, J.; Chaumette, F. "Visual servoing in robotics scheme using a camera/laser-stripe sensor," pp.743-750
- Nayar, S.K.; Nene, S.A.; Murase, H. "Subspace methods for robot vision," pp.750-758
- Kelly, R. "Robust asymptotically stable visual servoing of planar robots," pp.759-766
- Hashimoto, K.; Ebine, T.; Kimura, H. "Visual servoing with hand-eye manipulator-optimal control approach," pp.766-774

POSITION-BASED VISUAL SERVO

Wilson, W.J.; Williams Hulls, C.C.; Bell, G.S. "Relative end-effector control using Cartesian position based visual servoing," T-RA, vol.12, no.5, pp.684-696, Oct 1996

- Computer vision data are used to compute the pose of the camera d, R relative to the world frame.
- The error $e(t)$ is defined in the pose space, $d \in \mathbb{R}^3, R \in SO(3)$.
- The control signal $\xi = (v, \omega)$ is a camera body velocity.

If the goal pose is given by $d = 0, R = I$, the role of the computer vision system is to provide, in real time, a measurement of the pose error.



The following years...

Visual servo control is now a fairly popular research area within the robotics community. Topics of interest include (at least) the following:

- the search for better features
- switched control systems
- the use of novel imaging geometries
- consideration of dynamics
- investigation of novel sensors

All of these topics can be found in the pages of the Transactions.